Project Title: Development of Health Assessment Tools for Utility-Scale Wind Turbine Towers and Foundations

Contract Number: RD4-14       Milestone Number: 4       Report Date: January 23, 2018

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Congressional District (corporate office): Minnesota’s 3rd Congressional District

Congressional District (project locations): Minnesota’s 1st and 2nd Congressional Districts

MILESTONE REPORT

Executive Summary: In accordance with Exhibit A of the contract, the Scope of Work for RD4-14, the research goal is to develop a cost effective method for evaluating the current health and remaining useful life of a wind turbine’s tower and foundation.

The project objectives as stated in Exhibit A are as follows:

- Develop an economically viable and deployable system of sensors that can take the foundation and tower measurements required to perform a health and remaining useful life assessments.
- Develop a method of estimating the health and remaining useful life of the wind turbine tower and foundation using data from the developed sensor system and SCADA data from the wind turbine.
- Test the sensor system and estimation methods.

The project performance as stated in Exhibit A is planned to be achieved as follows:

- Accuracy and effectiveness of sensor system to be similar to the existing system on the Eolos wind turbine.
- Mobilization time of three days for installation of sensor system per turbine.
- Potential wind farm cost savings of $50,000 verses conventional sensing systems over the 20 year life of turbine.
- Development of a metric for estimating the remaining useful life of a wind turbine tower and foundation based on wind turbine historical SCADA data.
- Potential reduction of up to 40% in equipment costs per wind turbine compared to the cost of the system installed on the Eolos wind turbine.
Milestone 4 is complete. In milestone 4, the project team implemented the analysis methodology developed in milestone 3 to assess the health of the Eolos wind turbine and foundation.

**PROJECT FUNDING IS PROVIDED BY CUSTOMERS OF XCEL ENERGY THROUGH A GRANT FROM THE RENEWABLE DEVELOPMENT FUND.**

**Technical Progress:** The scope of work associated with milestone 4 has been completed. The following report describes the implementation of the analysis methodology.

### 1.0 Data Processing Methodology

The overall data processing methodology was described in the Milestone 3 report. This report details the implementation of this methodology to assess the health of the Eolos wind turbine and foundation. The entirety of the methodology was evaluated with the exception of an analysis of the measurement period required for an accurate assessment of remaining useful life. This assessment will require a full analysis of the 5+ years of data collected on the Eolos wind turbine, which necessitates a very significant amount of processing time. This analysis will be performed as part of the next task as the objective of Milestone 5 is to optimize the operation and accuracy of the health assessment system. Determining the minimum amount of analysis time required for an accurate assessment of structural health and remaining useful life will significantly optimize the structural health monitoring (SHM) system by minimizing the number of measurements required and the length of time the system needs to be deployed.

### 2.0 Data Acquisition

For Milestone 4, the research team investigated a number of combinations of sample rates and strain sensor quantities to optimize (minimize) the amount of data that need to be collected in order to accurately assess the health of a wind turbine foundation and estimate its remaining useful life. Table 1 shows the sample rates and number of strain gauges that were evaluated in this task using the Eolos dataset. Specifics of these analyses are provided below.

#### 2.1 SHM system optimization

To determine the minimum number of sensors and the lowest possible sample rate that still achieves the SHM goals, the research team made use of the foundation sensor system that has been collecting data nearly continuously since October 2011 on the University’s Eolos wind turbine. The Eolos system is a large, robust, non-mobile measurement system comprised of 20 uni-axial strain gauges, 10 thermocouples, and 1 bi-axial tilt sensor.

The research team started with the full Eolos sensor system of 20 strain gauges measuring at 20Hz and repeated the calculations multiple times with different subsets of sensors. The data were also down sampled, simulating a slower measurement rate. The results show the point at which the calculations of foundation rotational stiffness and damage equivalent load (DEL) deviate significantly from the calculations made with the full Eolos system of 20 strain gauges measuring at 20Hz. Detailed descriptions of the methods used to compute rotational stiffness and DEL can be found in the Milestone 3 report.
To conduct the system optimization, the research team selected a 6 hour dataset during which the wind turbine was operating in a variety of wind conditions. The time period of this data set was 2017-06-28 18:00:00 to 2017-06-29 00:00:00. Limiting the dataset to 6 hours allowed for a reduced processing time which made many iterations of analysis possible. Figure 1 below shows time-series plots of the wind speed and power output during the 6 hour period selected for analysis.

![Time series plots of wind speed and power output](image)

**Figure 1:** Time series of the windspeed (top) and power output (bottom) during the 6 hour measurements period of interest

Figure 2 shows histograms for the overturning moment, platform tilt, and power output during the 6 hour period. The power histogram shows a good range of output power occurred during the measurement window.
Figure 2: Histogram of overturning moment, platform tilt, and power output

Table 1: Test Matrix showing the number of sensors and measurement rates that were evaluated.

<table>
<thead>
<tr>
<th>Number of Strain Gauges</th>
<th>20Hz</th>
<th>5Hz</th>
<th>1Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

The rotational stiffness and DEL calculations that resulted from this analysis are shown in the following tables. Details of the DEL calculations are given in section 4.
Table 2: DEL matrix results for various measurement rates and numbers of strain gauges

<table>
<thead>
<tr>
<th>Number of Strain Gauges</th>
<th>Measurement Rate</th>
<th>20Hz</th>
<th>5Hz</th>
<th>1Hz</th>
<th>20Hz vs 5Hz</th>
<th>20Hz vs 1Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>5214 kN-m</td>
<td>5178 kN-m</td>
<td>5028 kN-m</td>
<td>0.68%</td>
<td>3.56%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5403 kN-m</td>
<td>5371 kN-m</td>
<td>5171 kN-m</td>
<td>0.58%</td>
<td>4.29%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5705 kN-m</td>
<td>5650 kN-m</td>
<td>5478 kN-m</td>
<td>0.98%</td>
<td>3.99%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5404 kN-m</td>
<td>5415 kN-m</td>
<td>5267 kN-m</td>
<td>0.20%</td>
<td>2.52%</td>
</tr>
</tbody>
</table>

Table 3: Rotational stiffness results for various measurement rates and numbers of strain gauges

<table>
<thead>
<tr>
<th>Number of Strain Gauges</th>
<th>Measurement Rate</th>
<th>20Hz</th>
<th>5Hz</th>
<th>1Hz</th>
<th>20Hz vs 5Hz</th>
<th>20Hz vs 1Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>93.3 GNm/rad</td>
<td>93.4 GNm/rad</td>
<td>93.3 GNm/rad</td>
<td>0.11%</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>93.2 GNm/rad</td>
<td>93.2 GNm/rad</td>
<td>93.1 GNm/rad</td>
<td>0.05%</td>
<td>0.01%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>92.7 GNm/rad</td>
<td>92.8 GNm/rad</td>
<td>92.8 GNm/rad</td>
<td>0.08%</td>
<td>0.08%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>85.1 GNm/rad</td>
<td>85.2 GNm/rad</td>
<td>85.1 GNm/rad</td>
<td>0.12%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Table 4: Comparison of DEL and Stiffness computed with both RDF and Eolos systems

<table>
<thead>
<tr>
<th>Measurement Rate = 20Hz</th>
<th>RDF</th>
<th>Eolos</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEL (kN-m)</td>
<td>5705.7 kNm</td>
<td>4774.1 kNm</td>
</tr>
<tr>
<td>Stiffness (GN-M/rad)</td>
<td>92.73 GNm/rad</td>
<td>86.0 GNm/rad</td>
</tr>
</tbody>
</table>

Several conclusions can be made from the sensor optimization process. Three strain gauges can adequately determine the DEL and stiffness of the foundation. However, two sensors appear to under predict the foundation stiffness as compared to using 20, 4, and 3 sensors. In addition, a 5 Hz sampling rate gives a very similar value for DEL as compared with 20 Hz and may be an adequate sampling rate.
The optimal sampling rate is discussed further in the next section where a frequency spectrum of the strain gauge measurements is analyzed.

A system of 20 sensors provides a more robust estimate of the maximum strain. The number of unique estimates for the maximum strain in the tower at each time step is given by \( n(n-1)/2 \) where \( n \) is the number of sensors. (Note: only one sensor out of two that are located 180 degrees apart is used in the analysis and therefore this equation was modified depending on the sensor spacing.) For 20 sensors this is equal to 180 unique solutions which provide a robust estimate in an average calculation. 3 sensors provide 3 unique solutions and therefore a maximum strain estimate with greater uncertainty. This likely explains why the DEL estimate increases with decreasing number of sensors, except in the 2 sensor case where the strain amplitude is under predicted.

### 2.2 Sample Rate Optimization

Following the comparison of the stiffness and the fatigue DEL at different sample rates, the optimized sample rate for data acquisition of the tiltmeter and strain gauges was confirmed by investigating the frequency content of measurements taken at a relatively high sampling rate of 20 Hz.

Figure 3 gives the averaged energy spectrum of strain gauge data taken at 20 Hz and windowed over 600 seconds for the 6 hour time series data. The peak highlighted at 0.338 Hz by the red circle indicates the tower natural frequency. A second prominent peak can be seen around 0.8 Hz which corresponds with the blade passing frequency. Some broadening of the peak is due to the rotor frequency changing during the measurement period. The figure also shows a flattening of the strain root mean square (RMS) around 5 Hz. RMS of the strain is a representative value of how much energy is present for a specific frequency range and is defined as \( \text{RMS} = \sqrt{\frac{A^2}{2}} \) for a sine wave where \( A \) is the amplitude of the wave.

Integrating the area under the spectra gives a representation of how much energy exists for a specific range of frequencies. Integrating from 0 to 2 Hz and normalizing by the total area gave 82% of the signal energy, similarly, integrating over the frequency ranges 0 to 4 Hz and 0 to 5 Hz gave 94% and 95% signal energy, respectively. This difference in signal energy is also represented in the DEL calculations in the above tables where the slower sampling rates generally predict less DEL as compared to the 20 Hz sample rate. This difference is on the order of 1% for 5 Hz compared with 20 Hz and 4% for 1 Hz compared with 20 Hz.

In conclusion, 10 Hz will be the sampling rate used. This results from a combination of the Nyquist rate, where the maximum frequency of interest is 5 Hz, and the analysis of the DEL testing.
2.3 Sensor Number Optimization

To confirm the minimum number of sensors determined by the stiffness and fatigue DEL comparison outlined above, we compared the maximum strain in the tower computed using the full set of 20 strain gauges and the maximum strain in the tower computed with subsets of 4, 3, and 2 strain gauges. Figure 4 below demonstrates the difference in maximum strain estimates using different numbers of sensors. While the solutions for maximum strain using 20, 4, and 3 sensors track each other nicely, there is slightly more noise in the data with fewer sensors which likely explains the increase DEL estimate for the 3 and 4 sensor cases vs using 20 sensors. The 2 sensor case consistently under predicts the maximum strain.
2.4 Temperature Correction Method comparison

The RDF sensor system and the Eolos sensor system use different methods to correct for the thermal expansion of the steel. The RDF system makes use of tee-rosette strain gauges which use orthogonally mounted gauges to compensate for thermal expansion, while the Eolos system uses measurements of the steel temperature to apply a computed correction factor. According to beam bending theory, the average of all measured strains at any given time should be zero. Computing the average strain from all gauges over a period of a few days allows for an analysis of temperature effects due to diurnal temperature fluctuations. The plot in figure 5 demonstrates that over a week of time in June, both methods of temperature compensation produced similar results. On a typical day, both systems showed an average strain that deviated from zero by about ±5 microstrain with some days showing an error as high as ±10 microstrain.
2.5 Data Processing: Engineering Units and Offsets

2.5.1 Strain Gauge

To remove the offsets that result from strain gauge installation and the eccentric loading applied to the tower by the rotor mass, new offsets were computed for both the RDF and Eolos systems using a yaw of the wind turbine. The table below lists the offsets that are applied to the strain gauge data before any further analysis is performed.

<table>
<thead>
<tr>
<th>Strain Gauge</th>
<th>Offset (micro-strain)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eolos 1</td>
<td>-6.0622</td>
</tr>
<tr>
<td>Eolos 2</td>
<td>-6.0959</td>
</tr>
<tr>
<td>Eolos 3</td>
<td>-9.8434</td>
</tr>
<tr>
<td>Eolos 4</td>
<td>+6.7444</td>
</tr>
<tr>
<td>Eolos 5</td>
<td>+19.7514</td>
</tr>
</tbody>
</table>

Figure 5: Top, 1 min. average strain for the Eolos and RDF systems. Bottom, temperature of the steel around the tower
Once the offsets have been applied, the strain and tilt data is filtered to remove any data spikes using a median filter.

The maximum strain, \( \varepsilon \), in the tower shell can be computed from any subset of at least 2 strain gauges as long as they are not mounted 180 degrees apart on the tower shell. Appendix A of Milestone 3 detailed a method of determining the maximum tower strain using an iterative solver method. As iterative solvers can be very resource intensive for computers, the research team developed a new method of maximum strain interpolation that does not require an iterative algorithm. Equation 1 shows the calculation of the location and value of maximum strain for an example case using two strain gauges located 90 degrees apart on the exterior of the tower shell. A derivation of this equation can be found in the Appendix of this report. Equation 1 is as follows,

\[
\alpha_{\text{max}} = \arctan \left( \frac{\varepsilon_2 - \cos(\alpha_2)}{\sin(\alpha_2)} \right) \tag{1}
\]

where \( \alpha_{\text{max}} \) is the angular location of the maximum strain on the tower with respect to the location of strain gauge 1. \( \alpha_1 \) and \( \alpha_2 \) are the angular locations of the strain gauges 1 and 2. \( \alpha_1 \) location is set to 0 in order to solve the system of equations outlined in Appendix A. \( \alpha_2 \) is the location of strain gauge 2 relative to \( \alpha_1 \). Note: \( \alpha_{\text{max}} \) is later adjusted so that it is with respect to North. \( \varepsilon_1 \) and \( \varepsilon_2 \) are the values of strain recorded by strain gauges 1 and 2 respectively.
Using the location of the maximum strain, \( \alpha_{\text{max}} \) (location w.r.t. \( \alpha_1 \)), the value of the maximum strain can be computed using the following equation.

\[
\varepsilon_{\text{max}} = \frac{\varepsilon_1}{\cos(\alpha_{\text{max}})}
\]  

(2)

If \( \alpha_{\text{max}} \) is equal to 90 degrees, the system of equations is solved such that the second strain gauge location is used as a different angular reference point. If a system of more than two strain gauges is being used, an estimate of maximum strain can be computed using each subset of two strain gauges that are not located 180 degrees apart. Taking an average of each of these estimates gives a more accurate measure of the maximum strain on the tower by removing the small errors introduced by electrical noise or strain gauge misalignment. The table below shows number of possible pairs that can be used to interpolate the maximum strain on the tower for each sensor configuration tested in the task.

<table>
<thead>
<tr>
<th>Number of Strain Gauges</th>
<th>Number of Maximum Strain Estimations</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Once the maximum strain in the tower has been calculated, the tower stress, \( \sigma \), is calculated using Hooke's Law,

\[
\sigma = \varepsilon \times E
\]  

(3)

where \( E = 2.9 \times 10^7 \) psi and is the modulus of elasticity of the tower material.

The overturning moment, \( M \), applied on the tower is then calculated from beam bending theory where the moment is equal to the stress times the section modulus:

\[
M = \sigma \times S
\]  

(4)

where \( S \) is the section modulus which is computed from the thickness and diameter of the shell. Using the outer diameter, \( OD \), and the inner diameter, \( ID \), the section modulus is calculated by:

\[
S = \pi \times (OD^4 - ID^4)/(32 \times OD)
\]  

(5)

where \( OD = 163.74 \) in and \( ID = 160.66 \) in. This will result in a time series of tower moments.

In order to compare this data to power output, 20 Hz power output is interpolated from the 1 Hz SCADA available from the turbine control system. Figure 6 shows the maximum moment and power output time series for the 6 hour period using a 10 second moving average.
**Figure 6:** Top, overturning moment time series. Bottom, power output time series

Figure 7 is a histogram showing probability of occurrence of overturning moment and output power.
2.5.2 Tiltmeter Data Processing

Foundation tilt, \( R \), is calculated as the resultant magnitude of the x and y tilt measurements. Raw tiltmeter output voltage from each component is converted to degrees using the manufacturer supplied calibration scale factor. Before the tilt magnitude is calculated, an offset for each component is applied. This offset zeroes the sensor. The needed offsets were determined by taking the average of the maximum and minimum tilt, for each component, during a 360 degree nacelle rotation (yaw).

The table below shows the offsets that were computed for the tiltmeter after a yaw calibration was performed.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Offset</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.002519 degrees</td>
</tr>
<tr>
<td>Y</td>
<td>0.020499 degrees</td>
</tr>
</tbody>
</table>

3.0 SHM Analysis

To assess the health of the foundation, the research team looked at the behavior of the foundation over the 6 hour period of time selected.
A value for rotational stiffness was calculated and compared to the wind turbine manufacturer's specification. In this case, the minimum required rotational stiffness specified by the wind turbine manufacturer was 60 GN-m/rad. Rotational stiffness, $K_\theta$, is defined as the overturning moment divided by the amount of rotation, $R$, and can be expressed as follows,

$$K_\theta = \frac{M}{R} \quad (6)$$

Some of the calculations that are used to estimate the rotational stiffness are shown in the table below. These calculations were made using a sample rate of 5 Hz and 20 strain gauge sensors.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rotational stiffness</td>
<td>93.39 GNm/rad</td>
</tr>
<tr>
<td>95th Percentile rotational stiffness</td>
<td>173.44 GNm/rad</td>
</tr>
<tr>
<td>95% confidence rotational stiffness</td>
<td>93.06 – 93.71 GNm/rad</td>
</tr>
<tr>
<td>Minimum rotational stiffness</td>
<td>14.25 GNm/rad</td>
</tr>
</tbody>
</table>

Figure 8 gives a histogram of the foundation tilt vs. overturning moment where the data was averaged down to 1 sec intervals. We would expect to see a linear relationship between platform tilt and overturning moment, that is, increasing tilt with increasing moment. However, the density plot below shows some nonlinear behavior with what appears to be two separate relationships between tilt and moment. We will investigate this further to see what may be causing this. Some hypotheses are: temperature effects on the tilt sensor or wind direction specific response of the tilt sensor or wind turbine. Figure 9 gives time series of the platform tilt, wind direction, and power output. The platform tilt appears to follow a trend similar to wind direction, however, it is possible that temperature may also be involved. Unfortunately, the temperature of the tilt sensor during the period in question is unavailable. We plan to investigate this issue further by finding measurement periods where the temperature was relatively constant (e.g. due to cloud cover) but the wind direction varied.
Figure 8: Histogram of Platform tilt vs. overturning moment
4.0 Fatigue Loading Analysis

The research team analyzed the 6 hours of data from the Eolos turbine using the analysis method outlined in milestone report 3. We have come across a few issues with the planned analysis method and will be investigating alternative methods during milestone 5. The following is a discussion of the analysis methods used and issues that came up.

4.1 Data Processing

The fatigue analysis procedure used the calculated moments from the strain gauges, high resolution SCADA data, and tilt measurements. The following is the procedure followed:

a) Using the processed moment time series (at the sample rate being tested), find the local maxima and minima using a peak finding function.
b) Apply a rainflow counting algorithm to the resulting maxima and minima from a). This gives the full and half cycles of the periodic data and moment range and mean for each cycle. Half cycles are treated as full cycles in the analysis.

Results from rainflow counting function include:

The mean moment, $S_{m,i}$, is equal to the average of the minima and maxima of the $i^{th}$ cycle.
The moment range, $S_{r,i}$, is the difference between the maxima and minima of the $i^{th}$ cycle.

c) Compute the damage equivalent load using data from b). A minimum threshold for the moment range is used to throw out small scale oscillations. This threshold was set at 100 kN-m for this analysis but may be changed in the future.

The following equation is used to calculate the damage equivalent load:

$$S_{r,eq} = \left( \sum_{i=1}^{n} \left( \frac{S_{r,i} - [S_{m,eq}]}{S_{u} - S_{m,eq}} \right) \right)^{\frac{1}{m}}$$

Where:
- $S_{r,eq}$ is the damage equivalent load (computed)
- $n$ is the total number of cycles (half and full) (measured)
- $S_{r,i}$ is the moment range of the $i^{th}$ cycle (measured)
- $S_{u}$ is the ultimate design load (85509 kN-m) (known)
- $S_{m,eq}$ is the equivalent mean moment (15900 kN-m) (known)
- $S_{m,i}$ is the mean moment of the $i^{th}$ cycle (measured)
- $N_{eq}$ is the equivalent number of load cycles (2e7) (known)

d) Count the total number of cycles from the rainflow counter (half cycles are treated as full cycles)
e) Divide the number of cycles from d) by the total design cycles from the OEM design fatigue spectrum. This gives the ratio of measured number of cycles to total design cycles. Apply the ratio to each bin of cycles in the design spectrum and calculate the design DEL using the prorated cycles and corresponding design mean moment and range. The same equation used in c) is used here.

To this point the analysis method works as Intended. Results for the DEL in c) and e) are very similar.

f) From data set a) and b), determine the high resolution power output that occurred during each cycle. This is calculated as the average output power the cycle.
g) Group the paired power output and fatigue cycles into 25 kW output power bins.
h) Compute the average and standard deviation of mean moment and moment range within each power bin. Count the number of cycles that occurred in each bin.
i) For each bin, calculate the design average mean and design average range based on a 99.0% percentile.
j) Calculate the DEL using the equation from c) and the design average mean and range from each bin. Compare this value with the DEL computed in c).

It is this point in the data processing that we have discovered a few issues. The DEL calculated in step j) is significantly different from what was obtained in c). Further, the DEL in j) varies significantly based on the percentile used in i) and the moment range threshold used in c). Further, there does not appear to be much correlation between moment range and output power making this an inappropriate method for binning. The correlation coefficient between moment range and mean moment is 0.099 and the coefficient between moment range and output power is 0.127. Figure 9 also demonstrates the weak relationship of power output and moment range. The team will investigate alternative binning methods for steps g) through j). One method of interest is to bin by moment range within each power bin.

Steps k) through q) in the analysis procedure were not thoroughly investigated due to the problems we discovered with the process in steps g) through j) detailed above. Once an appropriate binning method for steps g) through j) is developed, we expect the rest of the data processing procedure to proceed without further problems.
Figure 10: Histogram of moment range vs output power
Conclusions

As a result of the implementation of the optimization analysis outlined in Milestone 3, it has been concluded that future measurements will be performed at a sample rate of 10Hz using 3 strain gauges. This sample rate and number of strain gauges provides estimations of DEL and stiffness that are very similar to the estimations provided by the full Eolos system of 20 strain gauges measuring at a rate of 20Hz.

Additional Milestones: Milestone number 5 is ready to begin. In this milestone, the team will optimize the operation and accuracy of the system.

Project Status: The project is currently on schedule. Milestone number 5 has a duration of 2 months and is projected to be completed March 2018.
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Appendix A
Derivation of Maximum Strain Interpolation Method

The values of the measured strain can be expressed with relation to the maximum strain in the tower by the following two expressions:

\[
\begin{align*}
\epsilon_1 &= \epsilon_{max} \cdot \cos(\alpha_{max} - \alpha_1) \\
\epsilon_2 &= \epsilon_{max} \cdot \cos(\alpha_{max} - \alpha_2)
\end{align*}
\]  

(A1)

Divide the two strain expressions:

\[
\frac{\epsilon_1}{\epsilon_2} = \frac{\cos(\alpha_{max} - \alpha_1)}{\cos(\alpha_{max} - \alpha_2)}
\]  

(A2)

Choose coordinate system for \( \alpha_1 \) and \( \alpha_2 \) such that \( \alpha_2 \) is relative to \( \alpha_1 \). This allows us to make \( \alpha_1 = 0 \). Now we can use a trigonometric identity to simplify the expression and solve for \( \alpha_{max} \). \( \alpha_{max} \) position is with respect to \( \alpha_1 \).

\[
\frac{\epsilon_1}{\epsilon_2} = \cos(\alpha_{max})
\]

(A3)

\[
\frac{\epsilon_2}{\epsilon_1} = \cos(\alpha_2) + \frac{\sin(\alpha_{max})}{\cos(\alpha_{max})} \sin(\alpha_2)
\]

(A4)

\[
\frac{\epsilon_2}{\epsilon_1} - \cos(\alpha_2) = \tan(\alpha_{max}) \sin(\alpha_2)
\]

(A5)

\[
\frac{\epsilon_2}{\epsilon_1} - \cos(\alpha_2) \sin(\alpha_2) = \tan(\alpha_{max})
\]

(A6)

\[
\alpha_{max} = \arctan \left( \frac{\epsilon_2}{\epsilon_1} \frac{\epsilon_2 - \cos(\alpha_2)}{\sin(\alpha_2)} \right)
\]

(A7)

Now use the value of \( \alpha_{max} \) to compute the value of \( \epsilon_{max} \).

\[
\epsilon_{max} = \frac{\epsilon_1}{\cos(\alpha_{max})}
\]

(A8)

Or

\[
\epsilon_{max} = \frac{\epsilon_2}{\cos(\alpha_{max} - \alpha_2)}
\]

(A9)